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Dynamic response of forced convective heat transfer from hot-film sensors to mercury. Part 2. Experiment

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The thermal response of hot-film anemometers to fluid velocity fluctuations is investigated for low-Prandtl-number fluids. A quartz-coated hot-film probe with a diameter of 0·002 inches and an aspect ratio of 20 was oscillated in a horizontal plane while immersed in a steadily rotating tank of mercury. The probe was oscillated sinusoidally from 2 to 1200 Hz with a vibrator. The amplitude of velocity fluctuation was regulated to about 20 % of the mean flow within a Peclet-number range of 0·1–1·0.

The findings concur with the theoretical results obtained numerically and published earlier by Malcolm & Verma (1973) and compare well with some results of other researchers. The results confirm that the sensitivity of the hot-film probes is inhibited at low Peclet numbers, even at quite low frequencies in liquid metals owing to their very low Prandtl numbers.

Two important effects are noticed:

(*a*) The amplitude of fluctuation is attenuated and the degree of attenuation depends upon a non-dimentional quantity $\alpha f/\overline{U}^2$, for the range of Peclet numbers considered, where α is the thermal diffusivity, f is the frequency of the fluctuations and $\overline{U}$ is the free-stream velocity, in compatible units. The amplitude is attenuated by 10 % and 90 % at $\alpha f/\overline{U}^2$ values of 0·02 and 4·0 respectively.

(*b*) There is a phase lag in the hot-film probe signal with respect to the true velocity of the fluctuation which is somewhat the same as that in potential flow at low frequencies, but is considerably higher than that in potential flow at higher frequencies. The measured lag does not level off asympotically at high frequencies as noted in the numerically obtained results for potential flow.

Corrections may be made to unsteady velocity measurements in low Prandtl number fluids to account for the above effects with some confidence depending upon the value of the $\alpha f/\overline{U}^2$ quantity. The results of the investigation are of more general interest, in that the hot-film sensor can be considered as a model of a long circular cylinder in a flow at low to moderate Peclet numbers.

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1. Introduction

When a hot body is immersed in a fluctuating or turbulent flow, maxima in the fluctuations of heat transfer from the surface of the body tend to lag behind maxima in the fluctuations of free-stream velocity as a result of thermal inertia in the boundary layer. At low Peclet numbers, the thermal boundary layer extends much further from the body than the viscous boundary layer. The temperature field therefore tends to be insensitive to sudden changes in the velocity field and does not adjust itself instantly to the velocity fluctuations. Since quartz-insulated hot-film anemometer probes have become available, it has been possible to measure these effects in liquid metals.

Many researchers have made accurate velocity and temperature measurements in liquid metals with the help of hot-film probes. However, deposition in time of the impurities on the probe surface has made it impossible to repeat the calibration curves. This means that a calibration curve cannot be constructed once and for all and be used repeatedly. One of the earliest efforts to compensate for this drift in the calibration was made by Sajben (1965), who suggested that, in spite of drift, the quantity

$$\left[\frac{1}{Nu(0)} - \frac{1}{Nu(Pe)}\right],$$

where $Nu(0)$ is the Nusselt number at zero flow velocity and $Nu(Pe)$ is the Nusselt number calculated for the measured flow, remains the same function of Pe. This idea was in effect a break-through as far as thermo-anemometry in liquid metals was concerned. Hence, once a calibration curve has been obtained, by plotting the bracketed term against Peclet number, Pe, it can be used to determine flow velocities by measuring $Nu(0)$ and $Nu(Pe)$ in any flow, providing that the thermal resistance of the impurities around the sensor is constant throughout these measurements. However, we found it not uncommon in our experiments with technical-grade mercury to encounter 30–40 mV drifts in anemometer bridge voltage in just 15 minutes of continuous submersion, which could mean an error of up to 0·03 in Pe. Further details of this experimental technique are found in Malcolm (1969).

Two of the most relevant works to our study were those of Lim & Sleicher (1974) and Sleicher & Lim (1973). In one (Lim & Sleicher 1974) they evaluated the frequency response of the heated element submerged in liquid metal by a perturbation method for Peclet numbers of up to 0·4. Velocity fluctuations were assumed small and second-order perturbations neglected. The Oseen approximation was made for the velocity field. The other paper (Sleicher & Lim 1973) presents the frequency response attenuation of hot-film anemometers for Peclet numbers of up to 4·0. A solution for the problem of heat transfer from an infinite cylinder normal to the potential flow of a fluid with a small sinusoidal fluctuation is presented. The velocity configuration of Malcolm & Verma (1973) was approximated to that of potential flow and the convection equation was solved numerically. The potential flow approximation, as compared with the Oseen approximation, was considered more reasonable for an intermediate range of Peclet numbers. Although the effect of amplitude of fluctuation was not reported, the numerical method of Malcolm & Verma (1973) was equally valid for large amplitudes of fluctuation.

Hoff (1969) found that a thin layer of gold or copper on the quartz insulation of the sensor minimized the drift. In his view, the drift is caused by a high concentration of

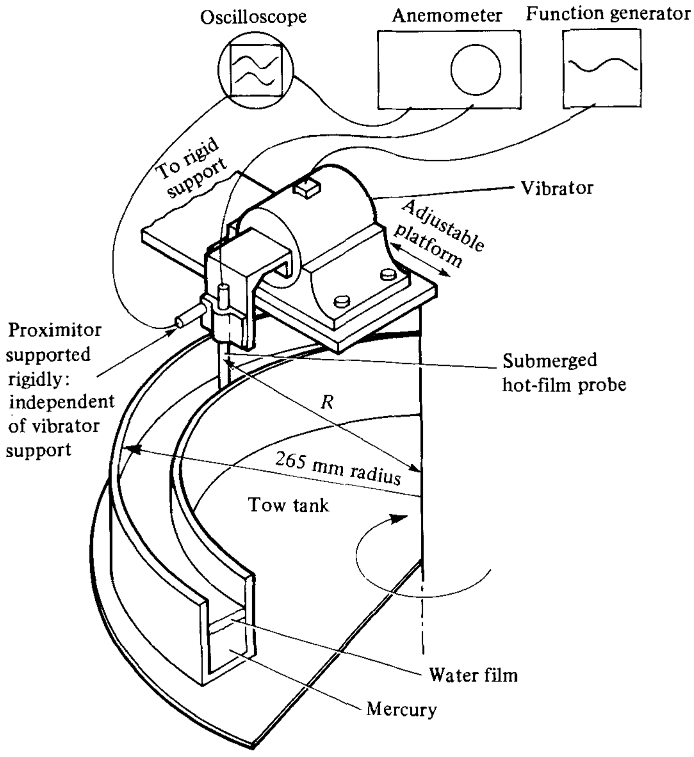

FIGURE 1. Apparatus and instrumentation.

impurities at the interface because of the non-wetting tendency of mercury with quartz. By depositing copper and letting mercury-vapour condense and form an amalgam on the sensor, two improvements could be made:

(i) wetting of the probe surface, thus preventing impurity concentration at the interface;

(ii) creation of a smooth surface over irregularities in the quartz layer.

An alternative procedure was used successfully by Robinson & Larsson (1973), in which the mercury flow system was totally enclosed and a thin coating of non-amalgamating metal, vanadium, was evaporated onto the quartz.

Several researchers have predicted a phase-lag and amplitude attenuation in hot-wire and hot-film probe measurements owing to the large thermal inertia of low-Prandtl-number fluid surrounding the sensor. Among them, Lighthill (1954), Lim & Sleicher (1974), Sleicher & Lim (1973), Strickland & Davis (1966) and Malcolm & Verma (1973) have theoretically computed the extent of the thermal inertia. However, owing to uncertainties in the true extent of phase lag and attentuation, the practice has been to ignore these effects. Our objective was to verify the theory experimentally by measuring the extent of amplitude attenuation and phase-lag in heat transfer signals (i.e. the bridge voltage) from a hot-film anemometer at different frequencies of flow fluctuation. The technique which we used is to measure the amplitude and phase angle of velocity fluctuations with a hot-film anemometer and another standard device such as a displacement transducer, then evaluate the amplitude ratio and phase difference as a function of the Peclet number and the frequency of fluctuation.

2. The experiment

2·1. *Sinusoidal flow simulation*

Apparatus was set up to simulate sinusoidal flow past a cylindrical hot-film sensor. Since low-Prandtl-number fluids were of interest, mercury was chosen for its relative ease in handling. A reliable and popular way of simulating pulsating flow is to develop a uniform flow equal to the mean flow while oscillating the body (in the present case the sensor) in the direction of flow with the desired alternating component of the flow. Uniform flow was obtained for these experiments by rotating an annular tank filled with mercury at constant speed. The tank and hot-film cylindrical sensor will now be described. The apparatus and instrumentation are depicted in figure 1.

An annular tank with two sections (the central and the annular compartments) was chosen over an ordinary circular tank because it requires less mercury, it reduces the exposed mercury surface, thus reducing evaporation, and it reduces the load on the supporting structure. The diameter of the tank was made as large as possible so that much of the vorticity created in the wake of the probe stem decayed in one revolution of the tank. Vertical shaft supporting the tank was coupled to an electric ratio motor with speed reduction in several stages, one of these stages being sprocket and chain transmission with interchangeable sprockets. Two rates of revolution was obtainable by changing sprockets on the ratio motor shaft. The radial distance at which the probe could be lowered into the tank was continuously adjustable from 38 mm to 260 mm giving a fluid velocity range of $0{\cdot}0098\,\mathrm{ms}^{-1}$ to $0{\cdot}098\,\mathrm{ms}^{-1}$ and a Pe range of 0·09 to 1·13.

Some basic conditions which must be satisfied in choosing the anemometer and the probe are:

(i) The maximum frequency response of the anemometer circuitry must be at least several octaves higher than the highest frequency of fluctuation.

(ii) The insulating material over the hot-film sensor should be as thin as possible so as not to induce any phase lag and attenuation owing to its own thermal capacity.

(iii) The aspect ratio of the sensor should be as large as possible so that one can make a valid comparison with the theoretical results for the infinite cylinder.

These conditions were satisfied by using commercially available sensors (1210-20HG TSI Incorporated hot film cylindrical probes) and anemometers (TSI Incorporated Model 1050 hot-film anemometer). The probes have a thin layer of quartz over the platinum film on a quartz core. The outer diameter of 0·051 mm and the length of 1·02 mm give an aspect ratio of 20.

The range of frequencies considered was 2–1200 Hz. For frequencies under 12 Hz a mechanical crank-type oscillator was used, above 12 Hz an electromagnetic vibrator was preferable for oscillating the probe and its support. The probe was lowered into the fluid with the aid of a light but rigid support structure.

2.2. *Instrumentation*

The quantities to be measured were:

(*a*) Heat-transfer rate (or the average Nusselt number) for the heated cylindrical sensor to mercury.

(*b*) Alternating component of probe velocity.

(*c*) Steady component of probe velocity (tangential velocity of fluid past the probe).

(*d*) Frequency of fluctuation of the probe.

In the measurement of the heat transfer rate, the sensor is maintained at a constant resistance, i.e. constant temperature, with the help of a feed-back controlled circuit and the potential drop across the sensor is measured. The average Nu is proportional to the square of the potential drop. The displacement transducer or the proximitor was used to measure the instantaneous alternating component of the probe velocity. The metal surface of the probe support (aluminium) was used as the target for the transducer, which was positioned close to the target with the help of a rigid frame. The steady component of the probe velocity was determined by measuring the radial distance R of the sensor from the centre of the tank and the angular speed of the tank N in revolutions per second

$$U = 2\pi NR \, \mathrm{ms}^{-1}.$$

The angular speed of the tank was measured with the help of a magnetic pick-up and an electronic counter. The time period of the fluctuation of probe was measured from the proximitor signal displayed on the oscilloscope.

2.3. *Procedure*

Before the actual experiment could proceed, the resistance coefficient of the sensor was determined by measuring its resistance at room temperature and at boiling point (100 °C). Also, the calibration of the proximitor with the help of a micrometer revealed that it had a linear response in the range 0·0025 mm – 1·016 mm and the transducer constant, Cp, was 0·0963 mm V^{-1}.

The next step in the experimental investigation was to perform a steady state calibration of the probe. Bridge voltage E was measured at zero velocity and at several other velocities and a calibration curve was obtained between E^* and Pe, where E^* is the E corresponding to $E(0) = 10{\cdot}25$ °V for any Pe. The value 10·25 V inherent in the definition of E^* is arbitrarily chosen. This $E^* \sim Pe$ relationship can subsequently be used in determining Pe for any flow provided a set of $E(Pe)$ and $E(0)$ is measured.

$$E^* = \left[\frac{1}{10{\cdot}25^2} - \left(\frac{1}{E^2(0)} - \frac{1}{E^2(Pe)}\right)\right]^{-0{\cdot}5}. \tag{1}$$

The steady-state correlation is useful in the interpretation of fluctuating flow observations. For several combinations of the tank's rate of rotation and the probe's radial distance (representing Pe numbers of 0·1, 0·25, 0·5, and 1·0), and for several frequencies of probe oscillation, the anemometer and proximitor signals were displayed simultaneously on the oscilloscope and recorded on the Polaroid film. The proximitor signal represents the distance measured in the direction of fluid velocity. Since the film only recorded the alternating component of the bridge voltage, it was necessary to measure the steady state bridge voltage, $E(Pe)$, also before the probe was allowed to oscillate so that the total voltage could be computed by adding the two. The amplitude of fluctuation could be monitored on the oscilloscope and was adjusted to about 20 % of the mean flow. The operating resistance of the sensor corresponding to an overheat ratio of 1·15 was set on the anemometer panel.

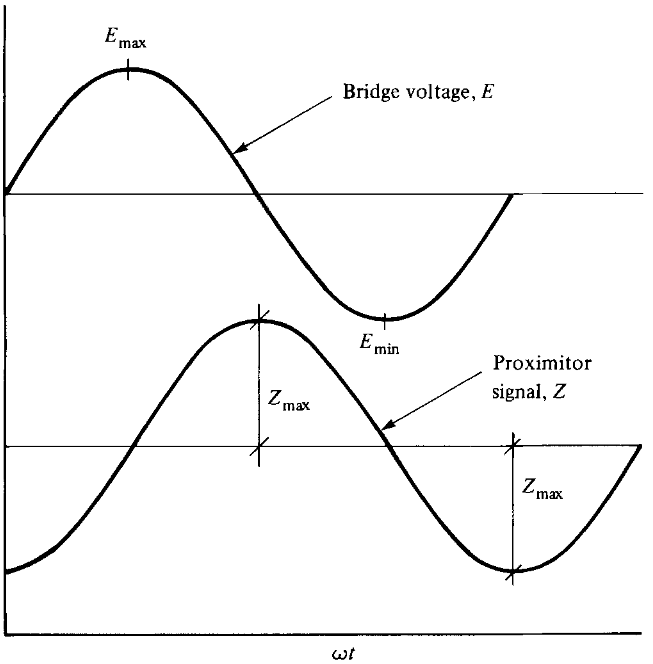

FIGURE 2. Typical bridge voltage, E, or Nu and displacement, Z, signals.

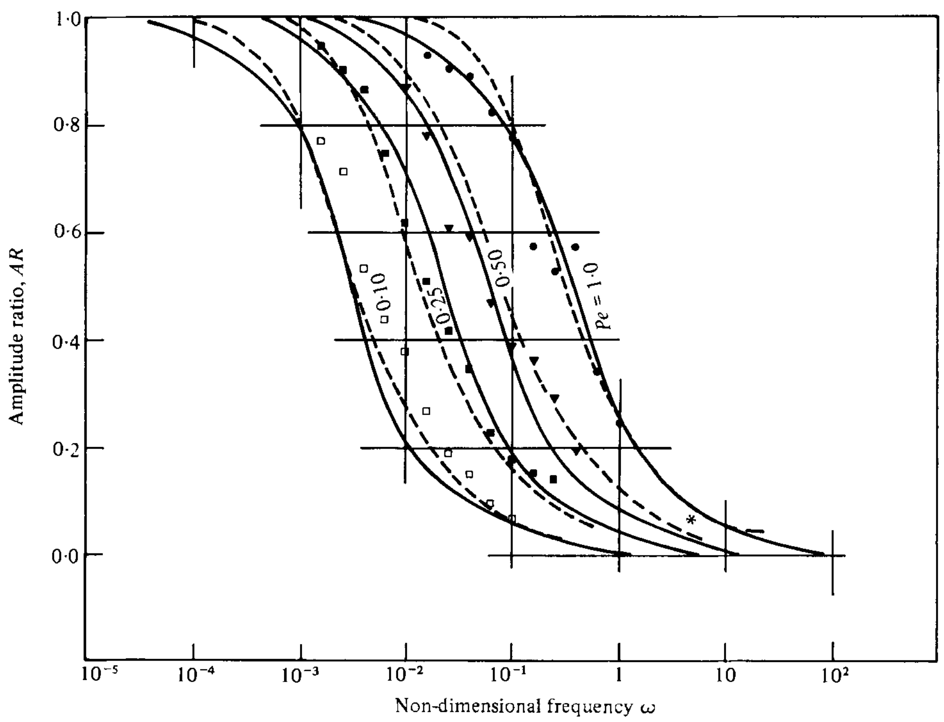

FIGURE 3. Experimental and calculated amplitude attenuation. Experimental results: ●, Pe = 1·02; ▼, 0·50; ■, 0·25; □, 0·10. ——, numerical solution of potential flow (Malcolm & Verma 1973); — — —, potential flow (Sleicher & Lim 1973); *, numerical solution of Navier–Stokes equation for Pe = 0·828.

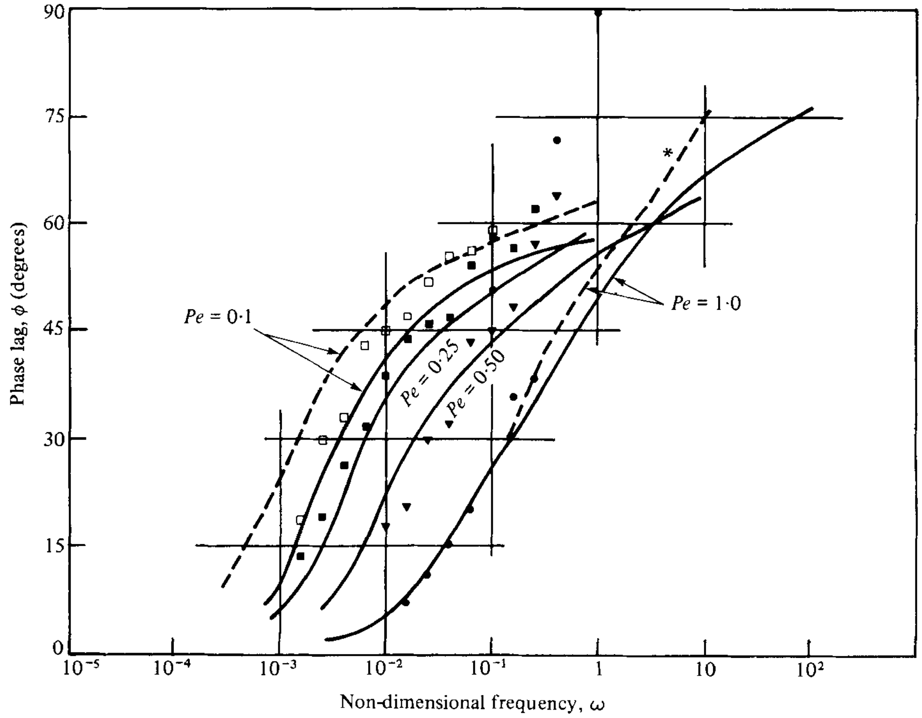

FIGURE 4. Experimental and calculated phase lag in heat-transfer response. See figure 3 for an explanation of the symbols.

3. The results

The calibration curve between Pe and E^* was found to have the following relationship for steady flows:

For $Pe < 0{\cdot}4$,

$$E^* = (105{\cdot}06 + 74{\cdot}48\,Pe)^{0{\cdot}5}, \tag{2}$$

and for $0{\cdot}4 \leqslant Pe \leqslant 1{\cdot}1$,

$$E^* = (106{\cdot}1 + 46{\cdot}78\,Pe^{0{\cdot}562})^{0{\cdot}5}. \tag{3}$$

With reference to figure 2, the maximum voltage E_{max} and the minimum voltage E_{min} (in one cycle of fluctuation) are obtained from the Polaroid photograph by assuming that the mean of the alternating component of the bridge voltage signal represents the steady state bridge voltage measured before the oscillations took place. The E_{max} and E_{min} thus obtained must be transformed to E^* using equation (1) and the $E(0)$ last measured. Pe_{max} and Pe_{min} as sensed by the inhibited hot-film probe are then derived from E^*_{max} and E^*_{min} as described in the appendix. The corresponding true Peclet numbers, on the other hand, are calculated from the proximitor signals recorded on the Polaroid film. This procedure is also described in the appendix. The maximum amplitude of fluctuation Z_{max} is measured from the photograph in terms of voltage and converted into metres (refer to figure 2).

$$Z_{max}(\mathrm{m}) = Z_{max}(\mathrm{V}) \times Cp/1000. \tag{4}$$

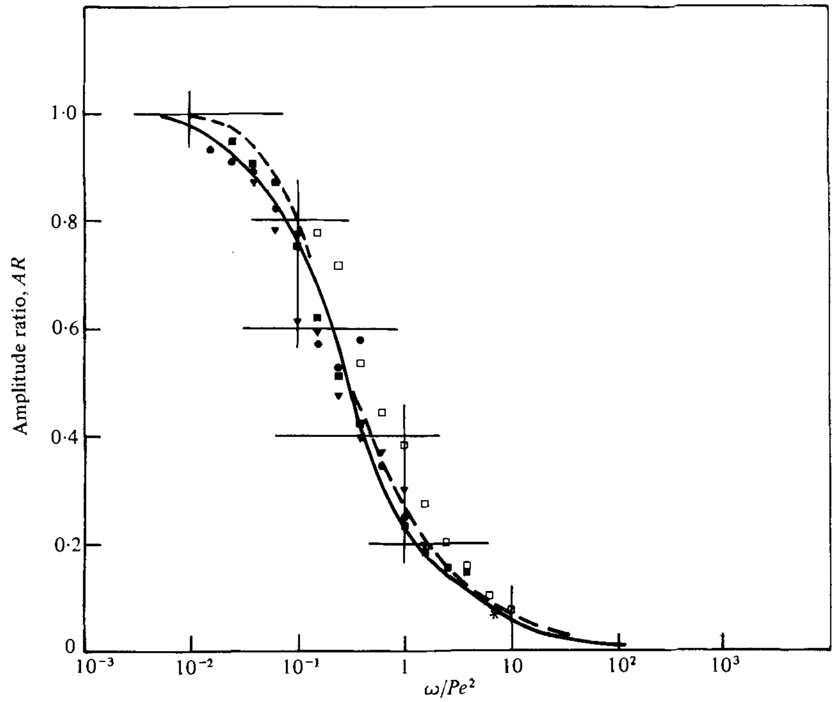

FIGURE 5. Experimental and calculated relationship between amplitude ratio and ω/Pe^2. See figure 3 for an explanation of the symbols.

From the appendix, the amplitude ratio, AR, of the actual instantaneous Nusselt number to its quasi-steady value is given as:

$$AR = \frac{(Pe_{\max} - Pe_{\min})_{\text{inhibited}}}{(Pe_{\max} - Pe_{\min})};$$

or

$$AR = \frac{(Pe_{\max} - Pe_{\min})_{\text{inhibited}}}{4\pi f Z_{\max} d/\alpha}, \tag{5}$$

where f is the frequency of flow fluctuation, d is the diameter of the hot film sensor and α is the thermal diffusivity.

The phase lag is computed by measuring the wavelength, the distance between the corresponding crests of the bridge and proximitor signals, and taking into account the fact that proximitor signals are 90 degrees out of phase with the true velocity function.

The results are tabulated in table 1. They are compared with the theoretical results of Malcolm & Verma (1973) and Sleicher & Lim (1973) in figures 3, 4 and 5. Figures 3 and 4 show the amplitude ratio and the phase lag in the heat transfer response and figure 5 shows the relationship between the amplitude ratio, AR, and $\omega/\overline{Pe}^2$, where ω is the non-dimensional frequency of fluctuation, $\pi d^2 f/2\alpha$, and $\overline{Pe}$ is the mean Peclet number. The relatively close agreement between the experimental values and those predicted earlier lends credence to our results, although the experimental phase lag is significantly greater than that predicted for $\overline{Pe} \sim 1$. From figure 5, it can be concluded that the amplitude of fluctuation of the Nu signals (proportional to inhibited Pe values)

Frequency		$\overline{Pe} = 0{\cdot}1$		$\overline{Pe} = 0{\cdot}25$		$\overline{Pe} = 0{\cdot}5$		$\overline{Pe} = 1{\cdot}02$	
f (Hz)	$\omega \times 10^3$ ($\omega = \pi f d^2/2\alpha$)	Amplitude Ratio AR	Phase lag ϕ	Amplitude Ratio AR	Phase lag ϕ	Amplitude Ratio AR	Phase lag ϕ	Amplitude Ratio AR	Phase lag ϕ
1·96	1·58	0·775	19°	0·951	14°				
3·112	2·51	0·714	30°	0·906	19°				
4·933	3·98	0·534	33°	0·872	27°				
7·818	6·31	0·441	43°	0·749	32°				
12·39	10	0·379	45°	0·62	39°	0·873	18°		
19·64	15·8	0·269	47°	0·51	44°	0·783	21°	0·929	7°
31·12	25·1	0·196	52°	0·418	46°	0·611	30°	0·911	11°
49·33	39·8	0·155	55°	0·346	47°	0·595	32°	0·893	15°
78·18	63·1	0·099	56°	0·227	54°	0·476	44°	0·82	21°
123·9	10^2	0·072	59°	0·117	59°	0·395	45°	0·778	51°
196·4	158			0·153	57°	0·361	49°	0·57	36°
311·2	251			0·142	62°	0·295	57°	0·525	38°
493·3	398					0·196	64°	0·576	72°
781·8	631							0·34	101°
1239	10^3							0·244	90°

TABLE 1. Experimental results for amplitude ratio and phase lag

is attenuated and the degree of attenuation depends upon the non-dimensional quantity $\omega/\overline{Pe}^2$. Reference to figure 5 shows that the amplitude ratio is 0·9 and 0·1 at $\omega/\overline{Pe}^2$ values of 0·032 and 6·3 respectively, (corresponding to $\alpha f/\overline{U}^2$ values 0·02 and 4·0) apparently irrespective of the Peclet number.

4. Concluding remarks

It is of considerable interest to mention some salient conclusions at this time. It appears that an attenuation in velocity of 10 % may be expected when $f = (0{\cdot}02/\alpha)\overline{U}^2$ Hz, and an attenuation 90 % when $f = (4{\cdot}0/\alpha)\overline{U}^2$ Hz, where $\overline{U}$ is the mean stream velocity and α is the thermal diffusivity of the fluid. The experiments covered the range of Peclet numbers of up to unity. Both amplitude attenuation and phase lag were evident and increased with frequency. The practical significance of these results in mercury flows lies in the fact that a typical mean velocity of 10 cm s^{-1} in the laboratory, a 10 % attenuation will take place at only 45 Hz, and a 90 % attenuation will take place at about 9000 Hz. This 10 % attenuation corresponds to a sensitivity to turbulence eddies of approximately 0·2 cm in size, or approximately equal to a typical sensor length. Thus, one must conclude that turbulence measurements in a typical experimental apparatus using mercury may be meaningless if the mean flow velocity is less than 10 cm s^{-1}. Even at $\overline{U} = 1$ m s^{-1}, the 10 % cut-off is at approximately 4500 Hz.

Appendix

Derivations of the apparent or inhibited $Pe_{\max}$ and $Pe_{\min}$ from the bridge voltage signal recorded on the oscilloscope photo, and of the true $Pe_{\max}$ and $Pe_{\min}$ from the proximitor signal are outlined as follows.

Apparent Pe

The mean Peclet number, $\overline{Pe}$, is computed from

$$\overline{Pe} = 2\pi NR\,d/\alpha. \tag{A 1}$$

Also

$$Pe_{\max} + Pe_{\min} = 2\overline{Pe}. \tag{A 2}$$

Let equations (2) and (3) from the text be, in general terms,

$$E^* = (A + B\,Pe^n)^{0\cdot5}, \tag{A 3}$$

where A and B are empirical constants. $E_{\max}$ and $E_{\min}$ are known, whereby $E^*_{\max}$ and $E^*_{\min}$ can be derived using equation (1). Thus,

$$E^*_{\max} - E^*_{\min} = (A + B\,Pe^n{}_{\max})^{0\cdot5} - (A + B\,Pe^n{}_{\min})^{0\cdot5}. \tag{A 4}$$

From equations (A 2) and (A 4), $Pe_{\max}$ and $Pe_{\min}$ can be computed using an iterative procedure. A good initial value for $Pe_{\min}$ to start with is

$$Pe_{\min} = \overline{Pe} - \frac{E^*_{\max} - E^*_{\min}}{2(dE^*/dPe)} \tag{A 5}$$

where

$$\frac{dE^*}{dPe} = \frac{nBPe^{n-1}}{2(A + BPe^n)^{0\cdot5}}. \tag{A 6}$$

True Pe

The maximum and minimum velocities (with respect to the mean flow), $Z'_{\max}$ and $Z'_{\min}$, of the probe during a cycle of fluctuation are computed as shown below:

$$Z'_{\max} = 2\pi f Z_{\max}, \tag{A 7}$$

$$Z'_{\min} = -2\pi f Z_{\max}; \tag{A 8}$$

$Z_{\max}$ is in metres. Therefore, by definition of Peclet number,

$$\begin{aligned}(Pe_{\max} - Pe_{\min})_{\text{true}} &= (Z'_{\max} - Z'_{\min})\, d/\alpha \\ &= 4\pi f Z_{\max}\, d/\alpha. \end{aligned} \tag{A 9}$$

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